

# A Low-Noise and Wide-Band Esaki Diode Amplifier with a Comparatively High Negative Conductance Diode at 1.3 Gc/s

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**Abstract**—This paper discusses the stability problem, output power, saturation level, and noise figure of Esaki diode amplifiers, and describes design considerations of the broadband circulator type amplifier with a large negative conductance diode.

An experimental amplifier with a diode which has a negative resistance of  $-25$  ohms is also described. The amplifier has a 3 dB bandwidth of 20 per cent, 18 dB gain, and a 3.6 dB noise figure including 0.3 dB insertion loss of the circulator. The output level for which the gain is 1 dB lower than the small signal gain is  $-17$  dBm. These experimental results are in fair agreement with those estimated theoretically.

## I. INTRODUCTION

THE MICROWAVE Esaki diode amplifier has advantages in its low power consumption, simplicity, and moderately low noise performance. However, its poor power handling capability, and electrical and mechanical fragileness, have been large disadvantages of the amplifier. These problems can be overcome without sacrificing its noise performance by using a large junction area diode. The large junction area diode also introduces a high negative conductance which is difficult to match with a 50 ohm system.

This paper describes design considerations of a circulator type amplifier with a high negative conductance diode, and an experimental amplifier with a diode which has a negative resistance of  $-25$  ohms. The amplifier achieved a 3 dB bandwidth of 20 per cent with an 18 dB reflection gain. When the circulator was replaced by a directional coupler, the bandwidth was increased to 27 per cent with a 19 dB reflection gain. The output level at which the gain is 1 dB lower than the small signal gain is  $-17$  dBm. The noise figure is 3.6 dB including 0.3 dB insertion loss of the circulator.

## II. STABILITY PROBLEM OF A WIDE-BAND ESAKI DIODE AMPLIFIER AT LOW MICROWAVE FREQUENCIES

A good Esaki diode has a negative conductance in the frequency range from dc to microwave frequencies and its gain bandwidth product is so large that no available circulator can cover the entire band, especially in a low microwave frequency range. Therefore, for a low noise wide-band amplifier, the essential problem is to utilize the maximum bandwidth of an available circulator

without causing instability or poor noise performance. Moreover, if one wants to use a high conductance diode, a practical transformer, which is needed to match the diode to a 50 ohm system, is also a frequency sensitive element and limits the amplifier stabilization.

Aron [1] has discussed the optimum design of  $LC$  networks for a wide-band amplifier with an ideal, frequency independent circulator and transformer. However, since the practical circulator and transformer have such serious effects on stability and consequently on bandwidth of a  $L$ -band amplifier, the idealized synthesis theory is difficult to apply to a practical amplifier design.

An equivalent circuit of the Esaki diode is shown in Fig. 1. The mounting inductance and stray capacitance are respectively included in the diode series inductance  $L$  and the package capacitance  $C_0$ , and the capacitance  $C_0$  is hereafter considered as a part of the external circuit. A normalized diode impedance  $z$  is given by the following equation:

$$z = a + jb^2\theta + \frac{1}{-1 + j\theta} \quad (1)$$

where

$$z = \frac{Z_d}{R} : \text{normalized diode impedance} \quad (2)$$

$$\theta = \omega CR = \frac{f}{f_b} : \text{normalized frequency} \quad (3)$$

$$a = \frac{r}{R} : \text{normalized series resistance} \quad (4)$$

$$b = \frac{1}{R} \sqrt{\frac{L}{C}} : \text{diode stability parameter} \quad (5)$$

$$f_b = \frac{1}{2\pi CR} : \text{gain bandwidth index of the diode} \quad (6)$$

$Z_d$ : diode impedance

$-R$ : reciprocal of the negative conductance

$C$ : junction capacitance

$r$ : series resistance

$L$ : series inductance

$\omega = 2\pi f$ : operating angular frequency

The function  $(z-a)$  is shown in Fig. 2 as a function of

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$\theta$  for various values of parameter  $b$ . Assuming that the signal frequency is much lower than the self-resonant and the resistive cutoff frequencies of the diode, the equivalent circuit in Fig. 1 can be simplified as shown in Fig. 3. The elements of the simplified equivalent circuit are given approximately by

$$\frac{1}{R'} \cong \frac{1}{R} \{1 + a + \theta^2(2b^2 - a)\} \quad (7)$$

$$C' \cong C \{1 + 2a - b^2 + \theta^2 b^2\} \quad a, b^2 \ll 1 \quad (8)$$

Let us take an example in order to illustrate how much effect we have to expect from a practical circulator and a transformer. Assuming

$$\begin{aligned} -R &= -33.3 \text{ ohms} \\ C &= 2.94 \text{ picofarads} \\ r &= 2.5 \text{ ohms} \\ L &= 0.52 \text{ nanohenries} \\ C_0 &= 1 \text{ picofarad,} \end{aligned}$$

we obtain from (1)–(6)

$$\begin{aligned} a &= 0.075 \\ b &= 0.40 \\ f_b &= 1.625 \text{ Gc/s} \\ \theta &= 0.80 \text{ at } 1.3 \text{ Gc/s} \end{aligned}$$

from (7) and (8) or Fig. 2

$$\begin{aligned} -R' &= -25.9 \text{ ohms} \\ C' &= 3.21 \text{ picofarads} \end{aligned}$$

Therefore, in order to obtain a 20 dB reflection gain at 1.3 Gc/s, a 21.2 ohm load impedance and a 3.58 nanohenry tuning inductance are required.

Let us for convenience call the amplifier circuit excluding the diode as the “external circuit” and denote its impedance  $Z_e$  as shown in Fig. 4. The external circuit usually contains a tuning circuit, a circulator, a signal source and a load, and a transformer, and a stabilizing circuit, if necessary. Stability of the whole amplifier can be analyzed by drawing a Nyquist diagram for the impedance  $z + Z_e/R$ . (Because the diode impedance contains a pole, a slightly modified Nyquist criterion of stability must be used.) Impedance loci are drawn in Fig. 5 for configurations with an ideal transformer (Case I) and a quarter-wave transformer (Case II), assuming ideal circulators for both cases. In Case I, the amplifier is stable and a 500 Mc/s (40 per cent) 3 dB bandwidth with a single tuned type peak gain of 20 dB can be expected. In Case II, however, the amplifier will oscillate around 1.5 Gc/s due to reactions from the quarter-wave transformer.

A stabilizing circuit, which is a series combination of a parallel resonant circuit and a resistor as shown in Fig. 6, can be used in order to suppress the oscillation. This circuit is susceptible near the signal frequency and

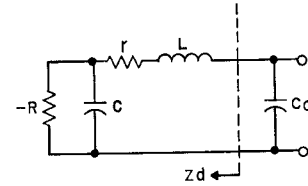


Fig. 1. Equivalent circuit of the Esaki diode.

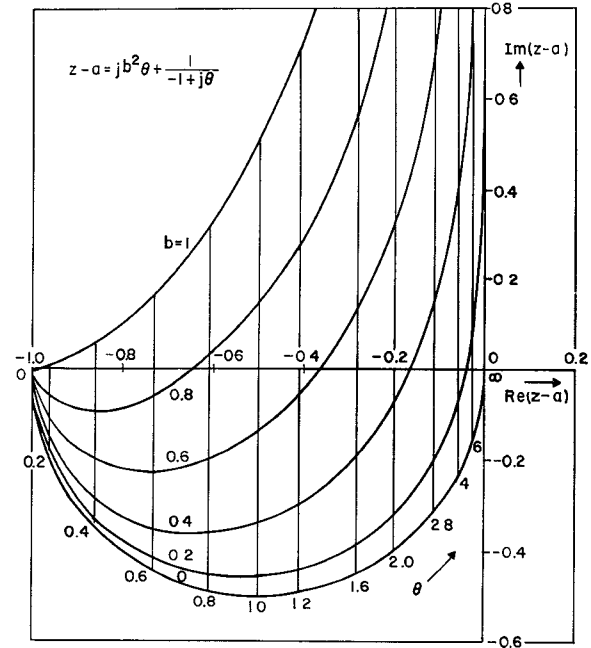


Fig. 2. Normalized impedance of the Esaki diode; variable  $\theta$ : normalized frequency, parameter  $b$ : diode stability parameter.

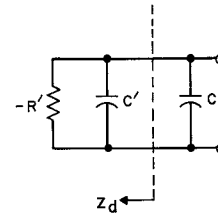


Fig. 3. Simplified equivalent circuit of the Esaki diode.

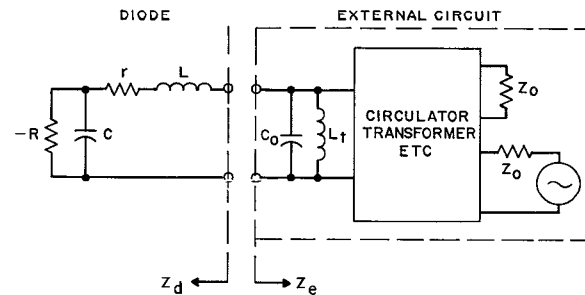


Fig. 4. Equivalent circuit of the amplifier with a diode.

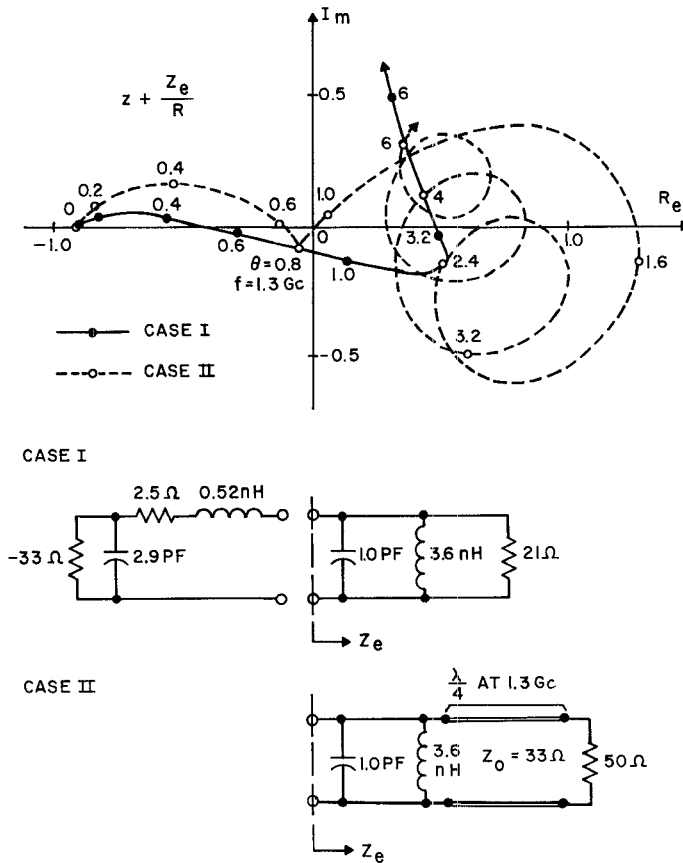


Fig. 5. Nyquist diagram of the amplifier for two external circuit configurations. (Case I and Case II.)

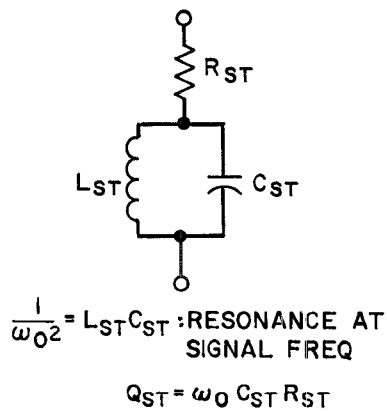


Fig. 6. A stabilizing circuit for the high negative conductance diode

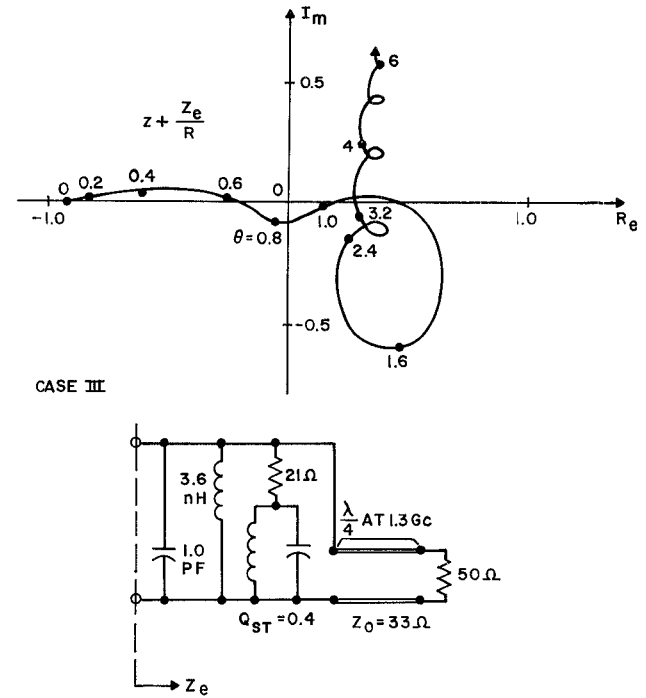


Fig. 7. Nyquist diagram of the amplifier with the stabilizing circuit (Case III).

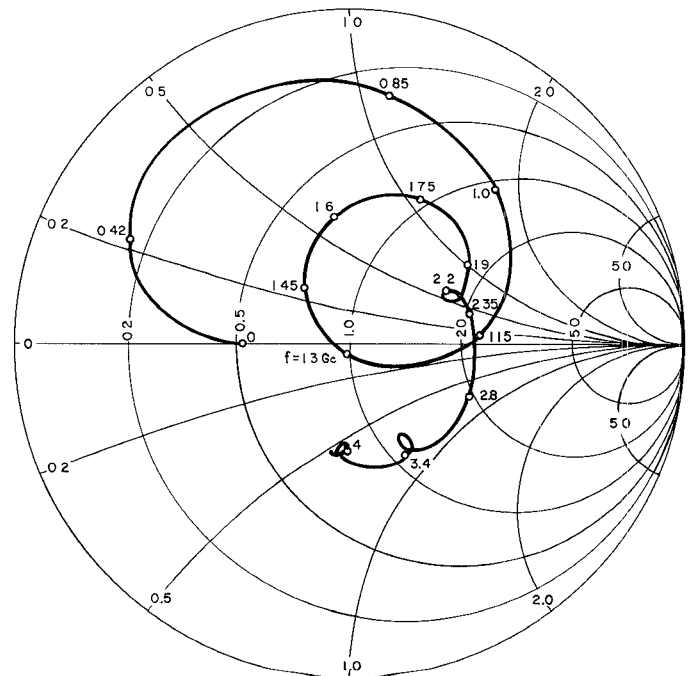
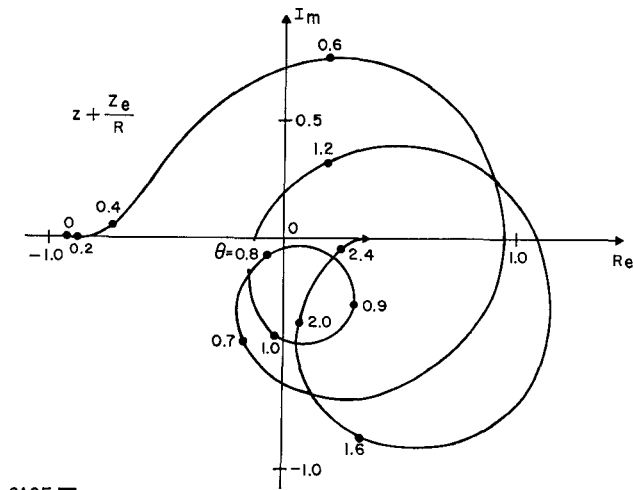


Fig. 8. Input impedance of the circulator, HL-1-1325, No. 101, Port 1.



CASE IV

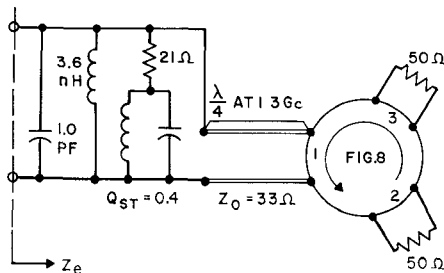
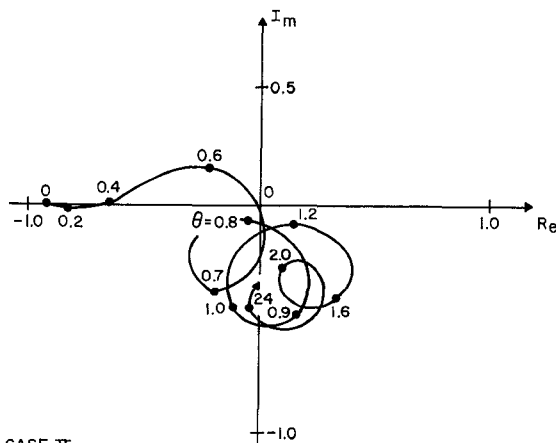


Fig. 9. Nyquist diagram of the amplifier with the circulator (Case IV).



CASE V

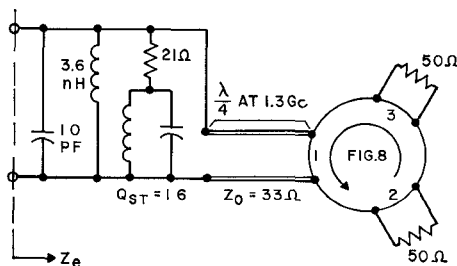


Fig. 10. Nyquist diagram of the amplifier with the circulator (Case V).

is conductive at frequencies far from the signal frequency. Since the parallel resonant circuit is resonant at the signal frequency, the resistor in the stabilizing circuit does not degrade the noise performance of the amplifier. As shown in Fig. 7, the amplifier with stabilizing circuit parameters of  $Q_{ST}=0.4$ ,  $R_{ST}=21$  ohms should be stable, and have a 600 Mc/s (45 per cent) 3 dB-bandwidth with a nearly double tuned type gain of 20 dB. The theoretical maximally flat bandwidth [2] calculated for the simplified double tuned equivalent circuit is 650 Mc/s (50 per cent); the amplifier bandwidth can be made to approach this by slightly reducing  $Q$  of the stabilizing circuit.

The bandwidth of circulators used in this experiment is much smaller than the above calculated bandwidth. This affects the amplifier performance very seriously even with a perfectly matched signal source and load. Figure 8 shows an example of a circulator impedance whose  $Q$  is about 2 centered around 1.3 Gc/s. The effect of this circulator impedance, which replaces the 50 ohm load impedance in Fig. 7 is rather disastrous. The Nyquist diagram for this case is shown in Fig. 9. The oscillation cannot be suppressed until the stabilizing circuit  $Q$  is increased to 1.6. The Nyquist diagram for this case,  $Q_{ST}=1.6$ ,  $R_{ST}=21$  ohms, is shown in Fig. 10. This diagram predicts two spurious gain peaks at 1.0 and 1.9 Gc/s, and a 100 Mc/s 3 dB-bandwidth near the center signal frequency.

In Section III we shall discuss simplified formulae for stability and bandwidth of a type of the amplifier illustrated in Fig. 10.

### III. STABILITY AND BANDWIDTH OF A SIMPLIFIED AMPLIFIER

An amplifier shown in Fig. 11(a) can be simplified as shown in Fig. 11(b) if the following conditions are satisfied:

- 1) The frequency range of interest is limited to near the signal frequency.
- 2) The circulator impedance is similar to that of a parallel resonant circuit with a 50 ohm load.
- 3) The negative resistance of the diode is considerably lower than 50 ohm.
- 4) The diode equivalent circuit can be simplified to that shown in Fig. 3.

In Fig. 11,  $R_L$  is the load resistance required to obtain a prescribed reflection gain and is given by the following relation:

$$\sqrt{PG} = \frac{R' + R_L}{R' - R_L} \quad (9)$$

where  $PG$  is the required power gain. The surge impedance  $Z_0$  of a quarter-wave transformer must satisfy

$$Z_0 = \sqrt{R_0 R_L} \quad (10)$$

where  $R_0$  is the surge impedance of the circulator (usually 50 ohms). The admittances  $y_1$  and  $y_2$  in Fig. 11(b)

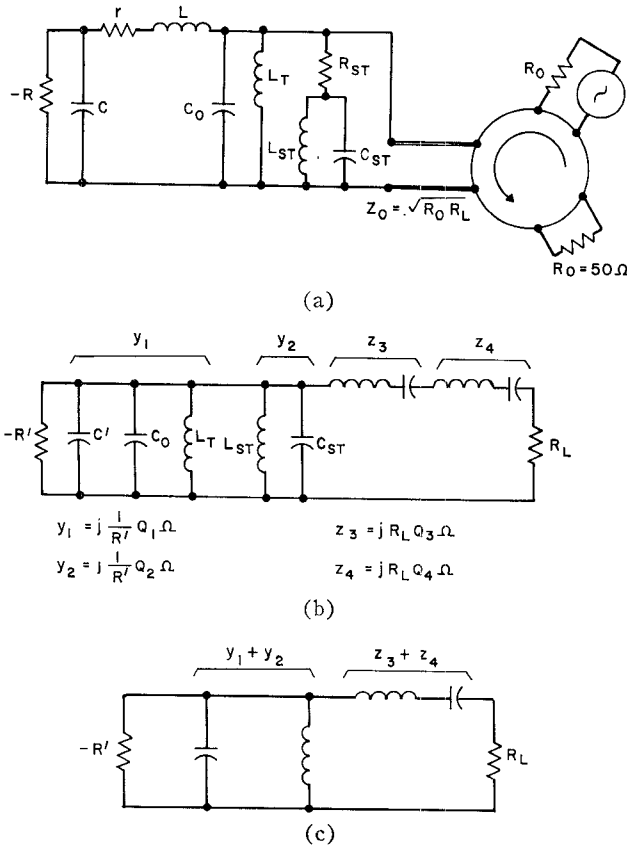


Fig. 11. Simplification of the amplifier equivalent circuit.

are in resonance at the signal frequency. The admittance  $y_1$  represents a diode susceptance with a stray capacitance and a tuning inductance, and  $y_2$  represents a susceptance of the stabilizing circuit. If we define  $Q$ 's of these admittances:

$$y_1 = j \frac{1}{R'} Q_1 \Omega \quad (11)$$

$$y_2 = j \frac{1}{R'} Q_2 \Omega \quad (12)$$

where

$$\Omega = \frac{f}{f_0} - \frac{f_0}{f}$$

and  $f_0$  is the signal center frequency,  $Q_1$  and  $Q_2$  are given by

$$Q_1 = \omega_0 (C' + C_0) R' \quad (11')$$

$$Q_2 = \omega_0 C_{ST} R' = Q_{ST} \frac{R'}{R_L} \quad (12')$$

The impedance  $z_3$  represents a series tuning effect of the quarter-wave transformer provided  $R_L < R_0$ , and  $z_4 + R_L$  represents the transformed impedance of the circulator. If we define  $Q$ 's of these impedances in the following equations:

$$z_3 = j R_L Q_3 \Omega \quad (13)$$

$$z_4 = j R_L Q_4 \Omega \quad (14)$$

$Q_3$  and  $Q_4$  can be obtained by

$$Q_3 = \frac{\pi}{4} \left( \sqrt{\frac{R_0}{R_L}} - \sqrt{\frac{R_L}{R_0}} \right) \quad (15)$$

$$Q_4 = Q_{CR} \quad \text{the } Q \text{ of the circulator.}$$

When we define  $Q_D$  and  $Q_L$  in the following equations:

$$Q_D = Q_1 + Q_2 \quad (16)$$

$$Q_L = Q_3 + Q_4 \quad (17)$$

the stability criterion of the circuit in Fig. 11(c) can be expressed by the following inequality:

$$1 > \frac{R_L}{R'} > \frac{R_L}{R'} \sqrt{\frac{Q_L}{Q_D}} \quad (18)$$

The maximally flat, relative 3 dB bandwidth  $(BW)_R$  of the amplifier shown in Fig. 11(b) is given in the literature [2].

$$(\sqrt[4]{PG} - 1)(BW)_R = \frac{\sqrt{2}}{Q_D} \sqrt{\frac{PG}{PG - 2}} \quad (19)$$

provided that

$$\frac{Q_L}{Q_D} = \frac{\sqrt[4]{PG} - 1}{\sqrt[4]{PG} + 1} \quad (20)$$

where

$$(BW)_R = \sqrt{\frac{f_2}{f_1}} - \sqrt{\frac{f_1}{f_2}} \cong \frac{f_2 - f_1}{f_0}$$

$f_1, f_2$  are the frequencies where the power gain is 3 dB less than the peak gain;  $PG$  is the peak power gain. For a numerical example, we assume:

$$-R' = -25.9 \text{ ohms}$$

$$C' = 3.21 \text{ picofarads}$$

$$C_0 = 1.0 \text{ picofarads.}$$

In order to obtain 20 dB gain at 1.3 Gc/s from (9) we obtain

$$R_L = 21.2 \text{ ohms.}$$

From (11')

$$Q_1 = 0.890 \quad \text{at } 1.3 \text{ Gc/s}$$

In order to obtain a 21.2 ohm resistance from a 50 ohm load, a quarter-wave transformer with  $Z_0 = 32.6$  ohms is required. From (15)

$$Q_3 = 0.69$$

If the circulator is ideal, i.e.,  $Q_4 = 0$ ,  $Q_D$  corresponding to the maximally flat characteristic is obtained from (20) as

$$Q_D = 1.34$$

Therefore, from (16)

$$Q_2 = 0.45$$

or

$$Q_{ST} = 0.37 \quad \text{if } R_{ST} = 21.2 \text{ ohms.}$$

The maximally flat percentage bandwidth is obtained from (19) as

$$(BW)_R = 0.49$$

If  $Q_4 = 0.5$  or  $2.0$ , the following values are calculated to obtain a 20 dB gain at 1.3 Gc/s:

Circulator $Q$ , $Q_4$	0	0.5	2.0
$Q_2$ required for a stable operation	0	0.30	1.80
$Q_2$ required for a max. flat gain	0.45	1.41	4.29
$(BW)_R$ for a max. flat gain	0.49	0.28	0.13

In the numerical examples of Section II, it was shown that for a stabilizing circuit having  $Q_2 = 0.49$  (i.e.,  $Q_{ST} = 0.4$ ,  $R_{ST} = 21$  ohms) one can expect a relative bandwidth  $(BW)_R$  of 0.45. It was also shown that in order to operate the amplifier in a stable condition with a practical circulator of  $Q_4 \sim 1.8$  and an ideal source and load condition, one needs a stabilizing circuit having  $Q_2$  greater than 1.95 (i.e.,  $Q_{ST} > 1.6$ ,  $R_{ST} = 21$  ohms). If one compares these results obtained by the elaborate method of Section II with those shown in the above table, one sees that the simplified formulae in this section can still predict approximate values of the bandwidth and stability for an amplifier with a single tuned type circulator.

#### IV. SATURATION LEVEL OF AN ESAKI DIODE AMPLIFIER

A major disadvantage of the Esaki diode amplifier has been its poor power handling capability. An easy way to improve the power handling capability is to use a diode capable of handling high voltage; however, this inevitably degrades the amplifier noise performance since a large shot noise contribution is unavoidable for such a diode. Another way to improve the saturation power level without sacrificing its noise performance is to use a large junction area diode. But the large junction area diode introduces a high negative conductance and makes the circuit design difficult.

Saturation in the Esaki diode amplifier is caused by nonlinearity in both the junction conductance and capacitance. Since the former predominates in the amplifier saturation characteristics, we shall restrict our discussion to the conductance nonlinearity.

Figure 12 shows an example of dc voltage-current characteristics of a gallium antimonide diode. In the useful negative conductance region, the measured curve is reasonably well approximated by the following simple equation:

$$I = I_{ex} + I_T(1 + \rho) \exp(1 - \rho) \quad (21)$$

where

$$I_T = \frac{V_2 - V_1}{R_{min}} \quad (22)$$

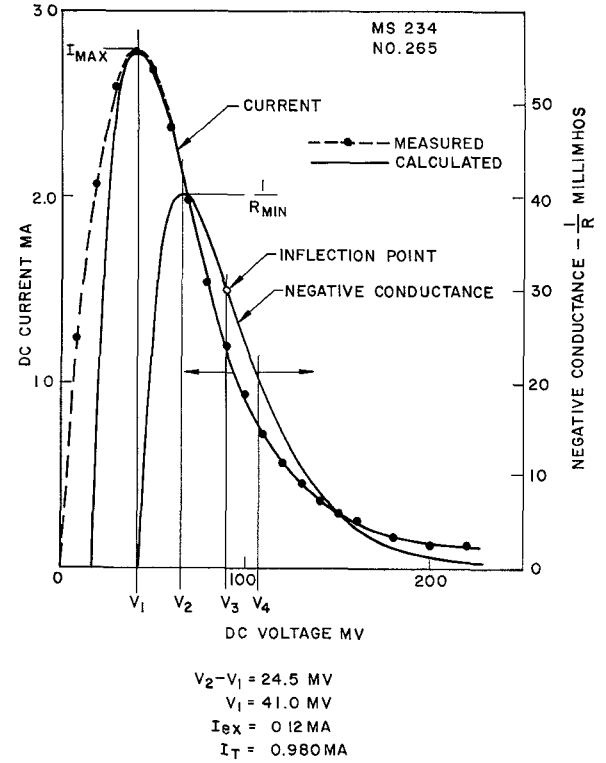


Fig. 12. Measured and calculated dc characteristics of the gallium antimonide diode MS234, No. 265.

$$\rho = \frac{V - V_1}{V_2 - V_1} \quad (23)$$

$I_{ex}$  is the excess current at valley point  
 $-R_{min}$  is the reciprocal of the maximum negative conductance,  
 $V_1$  is the peak voltage, and  
 $V_2$  is the voltage where the maximum negative conductance occurs.

From (21) we obtain the relation between  $I_T$  and the maximum dc current in the following equation:

$$I_{max} = I_{ex} + 2.72I_T \quad (24)$$

The negative conductance is obtained by differentiating (21) with respect to  $V$  as follows:

$$-\frac{1}{R} = \frac{dI}{dV} = -\frac{1}{R_{min}} \rho \exp(1 - \rho) \quad (25)$$

The maximum negative conductance is given by

$$-\left(\frac{1}{R}\right)_{max} = -\frac{1}{R_{min}}$$

at

$$V = V_2 \quad \text{or} \quad \rho_2 = 1.$$

An inflection point of the voltage-conductance curve occurs at

$$V = V_3 \equiv 2V_2 - V_1 \quad \text{or} \quad \rho_3 = 2$$

and the conductance value at this voltage is given by

$$-\frac{1}{R_{\text{inf}}} = -\frac{1}{1.36} \frac{1}{R_{\text{min}}} = -\frac{0.736}{R_{\text{min}}} \quad (26)$$

An effective negative conductance  $-g_{\text{eff}}$  for large signal operation can be obtained approximately by substituting the relation

$$V = V_0 + \tilde{V} \sin \omega t \quad (27)$$

into (21) and computing the fundamental component of the current. If we denote the small signal negative conductance by  $-g_0 (= -1/R)$ , the relative change in conductance,  $\sigma$ , due to saturation can be obtained from the following:

$$\begin{aligned} \sigma &= \frac{g_{\text{eff}} - g_0}{g_0} \\ &= \frac{1}{\rho_0} \left\{ (1 + \rho_0) \left( \frac{2I_1(\tilde{\rho})}{\tilde{\rho}} - 1 \right) \right. \\ &\quad \left. - (I_0(\tilde{\rho}) + I_2(\tilde{\rho}) - 1) \right\} \quad (28) \end{aligned}$$

where

$$\rho_0 \equiv \frac{V_0 - V_1}{V_2 - V_1} \quad (29)$$

$$\tilde{\rho} \equiv \frac{\tilde{V}}{V_2 - V_1} \quad (30)$$

$I_0(\tilde{\rho}), I_1(\tilde{\rho}), I_2(\tilde{\rho})$

are modified Bessel functions of the first kind.

Expanding (28) in power series, (28) yields

$$\sigma = \frac{\tilde{\rho}^2}{\rho_0} \left( \frac{\rho_0 - 2}{8} + \frac{\rho_0 - 4}{192} \tilde{\rho}^2 + \frac{\rho_0 - 6}{9216} \tilde{\rho}^4 + \dots \right) \quad (28')$$

In (28') we see that a high saturation level can be obtained at  $\rho_0 = 2$  or the inflection point of the voltage-conductance curve, because the first term of (28') disappears at this point.<sup>1</sup>

The small change in power gain  $\Delta PG$  due to the conductance change  $\sigma$  and the output power  $P_0$  of the reflection type amplifier can be explicitly obtained as follows:

$$\frac{\Delta PG}{PG} \cong \left( \sqrt{PG} - \frac{1}{\sqrt{PG}} \right) \sigma \quad (31)$$

$$\begin{aligned} P_0 &\cong \frac{1}{1 - \frac{1}{PG}} \frac{1}{R} |\tilde{V}|^2 \\ &= \frac{1}{1 - \frac{1}{PG}} \frac{(V_2 - V_1)^2}{R} |\tilde{\rho}|^2 \quad (32) \end{aligned}$$

In (32) the power  $(V_2 - V_1)^2/R$  is a figure of merit of the diode determining this power handling capability.

<sup>1</sup> The bias voltage for the highest saturation output does not always produce minimum intermodulation products, because the intermodulation products are closely related with the circuit condition for the harmonics.

### Numerical Example

Let us calculate the power handling capability of the diode whose characteristics are shown in Fig. 12. The measured dc characteristics of this diode can be represented analytically by (21) in the negative conductance region if one assumes the following constants:

$$\begin{aligned} V_2 - V_1 &= 24.5 \text{ millivolts} \\ V_1 &= 41.0 \text{ millivolts} \\ I_{\text{ex}} &= 0.12 \text{ milliamperes} \\ I_T &= 0.980 \text{ milliamperes.} \end{aligned}$$

In order to limit the effect of saturation to within 1 dB out of 20 dB power gain, the maximum permissible value of  $\sigma$  is calculated from (31) as

$$\sigma = 0.021$$

If one biases the diode at the inflection point of the voltage-conductance curve, i.e.,  $\rho_0 = 2$ , one obtains the following constants:

From (25) or Fig. 12,  $1/R = 30$  millimhos.

From (28')  $|\tilde{\rho}| = 1.41$ , corresponding to  $\sigma = 0.021$ .

Therefore, the 1 dB-compression output power level is obtained from (32) as

$$P_0 = -14.5 \text{ dBm.}$$

If one biases the diode at the maximum conductance point, i.e.,  $\rho_0 = 1$ , one obtains in the same way

$$P_0 = -24.0 \text{ dBm}$$

which is 9.5 dB lower than the level obtained by biasing at the inflection point.

If the diode is biased at  $\rho_0 = 2.62$ , where the shot noise contribution is minimum, as will be seen in Section V, from (28') one obtains  $\sigma$  value as high as 0.08 for the normalized applied RF voltage  $\tilde{\rho} = 2.32$ . At this  $\sigma$  value the gain increases as much as 5.4 dB from the small signal gain of 20 dB. Because of the existence of this gain peak below saturation, this bias condition is not suitable for a practical amplifier operation.

### V. NOISE FIGURE

The noise contribution of the Esaki diode itself to a reflection type amplifier is treated in literature [3]. If the noise sources of the Esaki diode can be represented by the noise generators in an equivalent circuit shown in Fig. 13, the optimum noise figure  $NF$  with infinite reflection gain is given by the following formulae:

$$NF(\text{dB}) = F_1(\text{dB}) + F_2(\text{dB}) \quad (33)$$

$$F_1(\text{dB}) = 10 \log_{10} \left( 1 + \frac{IR}{2kT_0/e} \right) \quad (34)$$

$$F_2(\text{dB}) = 10 \log_{10} \frac{1}{1 - a(1 + \theta^2)} \quad (35)$$

where

$I$ : diode dc current

$k$ : Boltzmann's constant,  $1.38 \times 10^{-23}$  Joules/°K

$e$ : electronic charge,  $1.60 \times 10^{-19}$  Coulombs

$T_0$ : standard temperature 290°K

$2kT_0/e = 50$  millivolts,

and  $\theta$  and  $a$  are the diode parameters given in (3) and (4).

The factor  $F_1(\text{dB})$  is the noise figure due to junction shot noise, and is a predominant factor for a low-noise amplifier. Since  $IR$  depends on junction material and the operating bias voltage, a diode for a low-noise amplifier must be made of appropriate materials and be operated in a suitable range of bias voltage. Figure 14 shows numerical values of  $F_1(\text{dB})$ . If the dc characteristic of the diode is well approximated by (21), the voltage  $V_4$ , where the minimum  $IR$  occurs, and the minimum value,  $(IR)_{\min}$ , are related to the other parameters by the following equations:

$$\rho_4 \equiv \frac{V_4 - V_1}{V_2 - V_4} = \frac{1}{1 - \frac{V_2 - V_1}{(IR)_{\min}}} \quad (36)$$

$$\frac{I_{\text{ex}}}{I_T} = \frac{\exp(1 - \rho_4)}{\rho_4 - 1} \quad (37)$$

If we solve (37) with respect to  $\rho_4$ , we obtain  $(IR)_{\min}$  from (36).

The second factor  $F_2(\text{dB})$  is a correction due to a finite series resistance and depends on the normalized frequency  $\theta$ . At the resistive cutoff frequency  $f_c$  given by

$$a(1 + \theta_c^2) = 1 \quad (38)$$

or

$$f_c = \frac{1}{2\pi RC} \sqrt{\frac{R}{r} - 1}, \quad (38')$$

$F_2(\text{dB})$  becomes infinite. In Fig. 15,  $F_2(\text{dB})$  is shown as a function of  $a$  and  $\theta$ . A nomograph to calculate  $\theta = \omega CR$  is included in Fig. 16 for the readers' convenience.

As a numerical example, the noise figure of the diode which was taken as the example in Section IV, is calculated. From (37) and (36),  $\rho_4$  and  $(IR)_{\min}$  are determined as

$$\begin{aligned} \rho_4 &= 2.62 \\ (IR)_{\min} &= 40 \text{ millivolts.} \end{aligned}$$

Substituting  $(IR)_{\min}$  into (34), or from Fig. 14(b),  $F_{1\min}$  is calculated to be 2.56 dB. From (25), the negative conductance at  $\rho = 2.62$  is calculated as  $-20.7$  millimhos. Assuming the following diode constants:

$$\begin{aligned} C &= 2.94 \text{ picofarads} \\ r &= 2.5 \text{ ohms,} \end{aligned}$$

and using (35) or Fig. 15(b), we calculate  $F_2$  as 0.57 dB. Therefore, the noise figure is  $NF = 3.1$  dB at  $\rho = 2.62$ . The true minimum  $NF$  occurs at a slightly lower bias

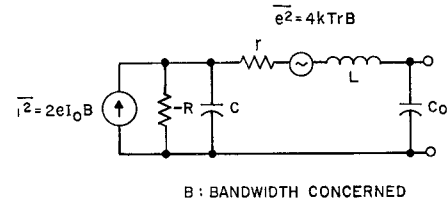


Fig. 13. Noise equivalent circuit of the Esaki diode.

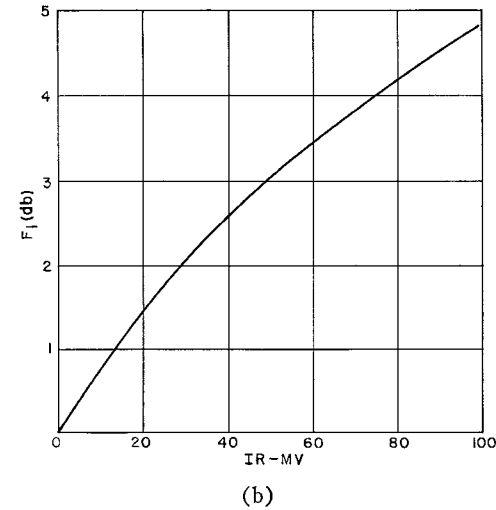
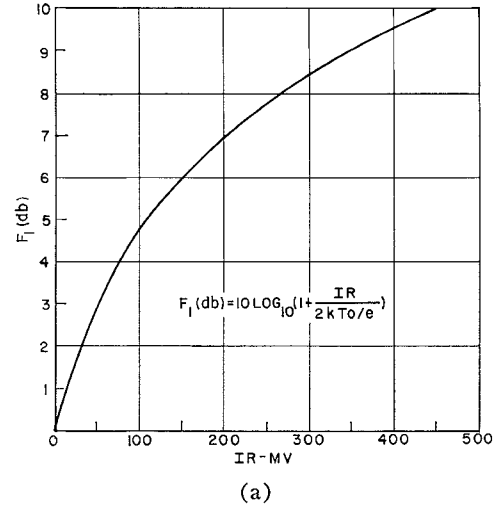


Fig. 14.  $F_1(\text{dB})$  as a function of  $IR$ ;  $NF = F_1 + F_2$ .

voltage than  $\rho = 2.62$ , since  $F_2$  becomes small for a high negative conductance value. But, as described in Section IV, the bias condition of  $\rho = 2.62$  is objectionable from a saturation point of view.

If we choose a bias condition of  $\rho = 2$ , from which we can expect the highest saturation level, the noise figure is calculated as follows:

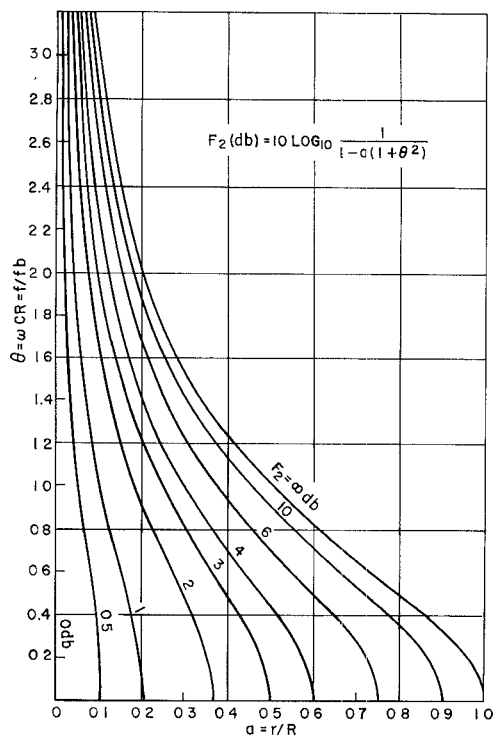
From (21) and (25) or Fig. 12

$$\begin{aligned} I &= 1.20 \text{ milliamperes} \\ 1/R &= 29.5 \text{ millimhos,} \end{aligned}$$

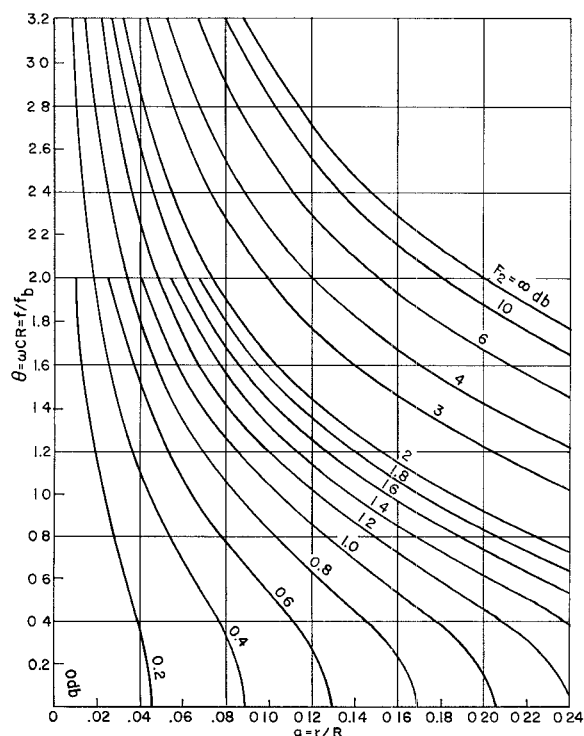
therefore

$$IR = 40.8 \text{ millivolts,}$$

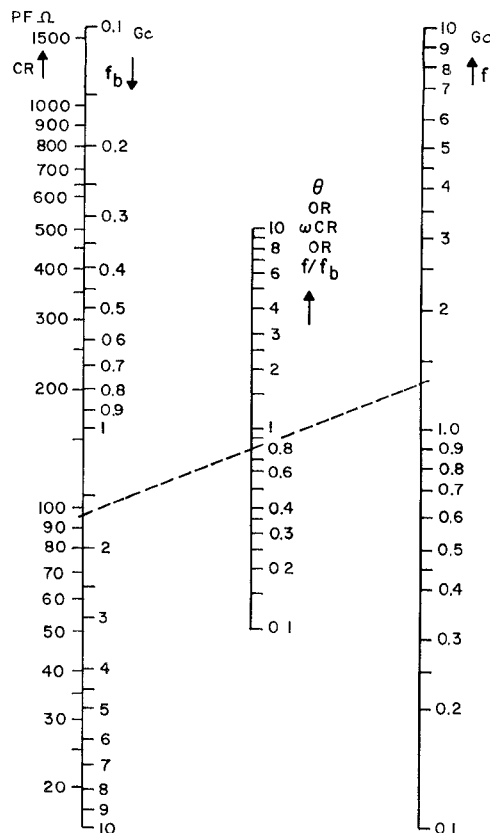




(a)



(b)

Fig. 15.  $F_2(\text{dB})$  as a function of  $\theta$  and  $a$ ;  $NF = F_1 + F_2$ .Fig. 16. Nomograph for computation of  $\theta = \omega CR$ .

and consequently

$$NF = 3.2 \text{ dB} \quad \text{at } \rho = 2$$

Since the difference in noise figures for the two operating conditions is only 0.1 dB, the latter operating bias ( $\rho = 2$ ) is more suitable for most practical operations.

## VI. EXPERIMENTAL AMPLIFIER

An experimental 1.3 Gc/s amplifier was built in a 50 ohm coaxial system by using the Esaki diode MS234, No. 265, manufactured by the Micro State Corp., Murray Hill, N. J. The dc characteristic of the diode measured by G. T. Orrok<sup>2</sup> is shown in Fig. 12. The total diode capacitance measured at 100 kc/s was 3.2 picofarads at  $V = 350$  milli-volts. The RF characteristics were also measured in a coaxial mount at 1.3 Gc/s and 2.0 Gc/s and the measured RF conductance and capacitance agree with those measured at dc and 100 kc/s. Hence, the equivalent circuit shown in Fig. 1 is a good representation of the diode. The measured values are shown as follows:

$C = 3.4$  picofarads at the valley point

$r = 2.7$  ohms including mount loss

$L = 0.5$  nanohenries including mounting inductance

$C_0 = 0.2$  picofarads.

<sup>2</sup> G. T. Orrok is a member of BELLCOMM Inc., Washington, D. C.

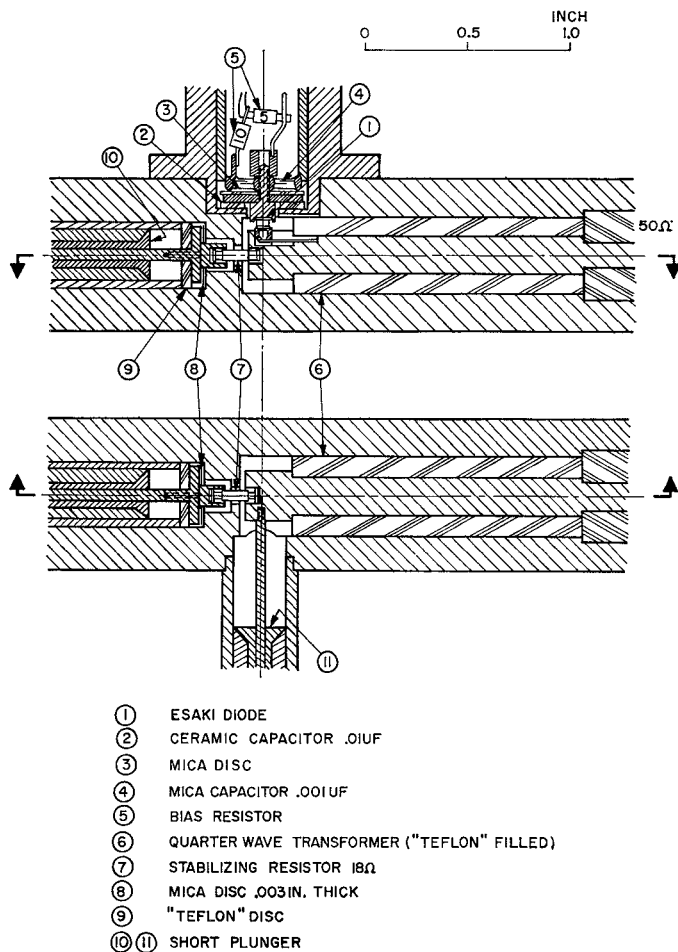
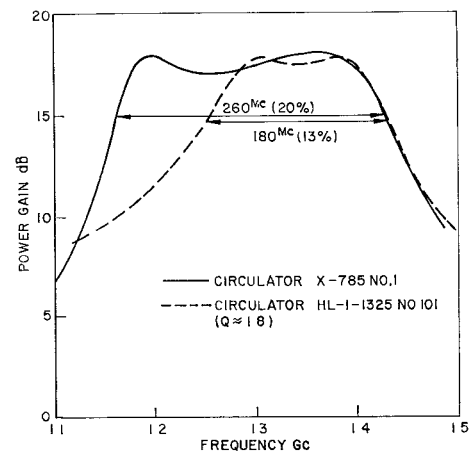


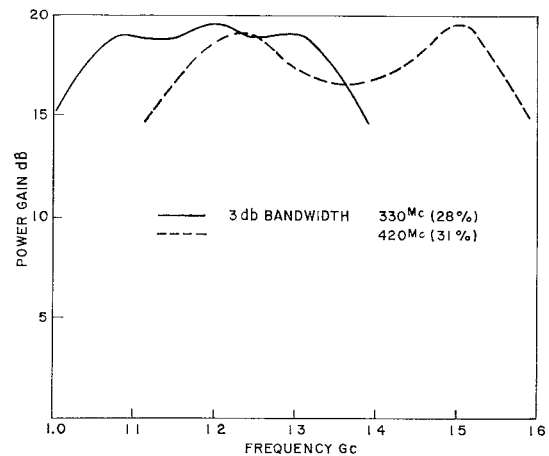
Fig. 17. Construction of an experimental amplifier.

Figure 17 shows the construction of the experimental amplifier. A quarter-wave transformer ⑥ transforms 18 ohms to 50 ohms at 1.3 Gc/s. A stabilizing circuit is constructed of a mica capacitor ⑧, an inductance ⑩ made of a shorted piece of line and a rod type resistor ⑦. In order to reduce the diode mounting inductance and to prevent spurious oscillations, a ceramic capacitor ② was used in the bias circuit. The bias circuit is almost identical to that used by K. Kurokawa [4], and provides for observation of the dc diode characteristics without removing the diode from the amplifier.

The measured gain characteristics are shown in Fig. 18. In Fig. 18(a), two curves show the gain characteristics of the same amplifier mount with two circulators of different bandwidth; X-785, No. 1 and HL-1-1325, No. 101, both made by Melabs Corp., Palo Alto, Calif. The former circulator has a double tuned type characteristic, while the latter has a single tuned type ( $Q=1.8$ ). The measured 3 dB bandwidth, i.e., 260 Mc/s (20 per cent) and 180 Mc/s (13 per cent), are completely limited by the circulator input impedance characteristics, and in order to minimize the effect of reflection from the load or signal source one had to limit the bandwidth to a half or two thirds of those shown in Fig. 18(a), dependent upon the maximum gain change which is tolerable for a particular application. Figure 18(b) shows the reflection gain character-



(a)



(b)

Fig. 18. Gain-frequency characteristics. (a) The amplifier with a circulator. (b) The amplifier with a directional coupler.

istics of the amplifier measured by using a directional coupler in place of the circulator. The measured 3 dB bandwidth of the amplifier tuned at a slightly lower frequency was 330 Mc/s (28 per cent). If a 3 dB hump is permissible, the bandwidth can be increased to 420 Mc/s (31 per cent). This hump is possibly due to small mismatches in the connectors and directional coupler.

The measured noise figures are shown in Fig. 19 for the amplifier with the X-785 circulator. The noise figure at 1.3 Gc/s is 3.6 dB including a 0.3 dB circulator insertion loss, and it stays less than 4.1 dB within the 3 dB bandwidth.

The saturation characteristics of the amplifier are shown in Fig. 20. As the input signal increases, the gain increases slightly at first and then decreases rapidly. This behavior shows that the bias voltage is slightly higher than the optimum bias voltage. The 1 dB compression output level for the amplifier with a 20 dB small signal gain, is  $-17$  dBm. If the bias voltage changes, the gain changes at a rate of  $-1.5$  dB/mV at a 20 dB gain point. The expected rate of the change is about  $-1$  dB/mV. (See Appendix) The experimental results for bandwidth, noise figures, and saturation, agree with the theoretical results shown in numerical examples of Sections III, IV, and V.

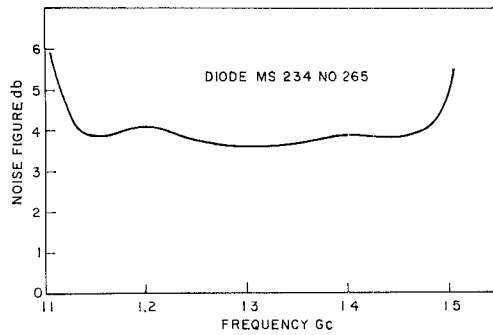


Fig. 19. Noise figure—frequency characteristics.

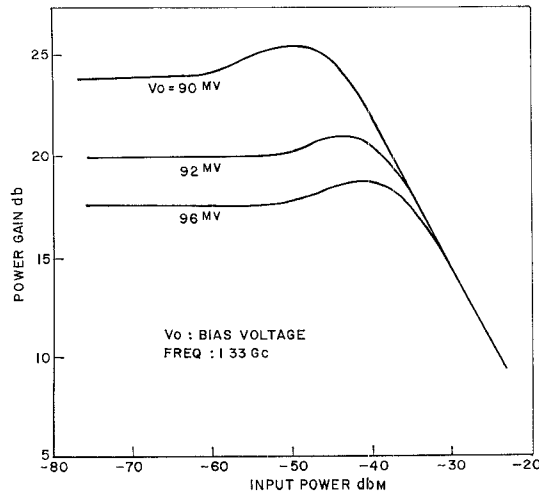


Fig. 20. Saturation characteristics.

## VII. CONCLUSION

Design considerations for low-noise wide-band circulator type Esaki diode amplifiers have been investigated. It was shown that the amplifier bandwidth is severely limited by the circulator characteristics, since a presently available diode has potentiality to obtain 20 dB gain over 1 Gc/s bandwidth. The output power saturation problem was also discussed. By using the dc current equation of the diode in its negative conductance region, the formulae for saturation output power were derived. In order to obtain a high saturation output, the bias voltage should be set near the inflection point, of the voltage-conductance curve. At this operating bias, the noise figure is slightly higher than the minimum obtainable one, however, the difference is usually insignificant compared with the improvement of the saturation characteristics. Theoretical curves to compute the noise figure are provided. A simple stabilizing circuit for a diode whose negative conductance is much higher than the conductance of a circulator was investigated and used in the experimental amplifier with a 25 ohm minimum negative resistance diode. The experimental amplifier with a circulator achieved a 3 dB bandwidth of 20 per cent with an 18 dB gain. The output level where the gain is 1 dB less than the small signal gain was -17 dBm. The noise figure was 3.6 dB including a 0.3 dB

circulator insertion loss. These results agree with the theoretical results within a reasonable accuracy.

## ACKNOWLEDGMENT

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## APPENDIX

### GAIN SENSITIVITY FOR THE DC BIAS VOLTAGE

From (21),

$$\Delta\left(\frac{dI}{dV}\right) = \frac{d^2I}{dV^2} \Delta V$$

$$= -\frac{1}{R_{\min}} \frac{\Delta V}{V_2 - V_1} (1 - \rho) \exp(1 - \rho) \quad (39)$$

If we define a relative conductance change  $\sigma'$  as

$$\sigma' \equiv \frac{\Delta\left(\frac{dI}{dV}\right)}{\frac{dI}{dV}} \quad (40)$$

we obtain the following relation:

$$\sigma' = \frac{1 - \rho}{\rho} \frac{\Delta V}{V_2 - V_1} \quad (41)$$

Therefore, the gain sensitivity for the dc bias voltage is the lowest at  $\rho=1$  (at the maximum negative conductance point) and becomes higher as the bias voltage increases.

### Numerical Example

For the same diode that was taken as an example in Sections IV and V, the value of  $\sigma'$  which changes the gain by 0.1 dB out of 20 dB gain, is obtained by (31) as

$$\sigma' = 0.0023$$

From (41) this  $\sigma'$  value corresponds to

$$\Delta V = 0.113 \text{ millivolts.}$$

Therefore, the rate of change of the gain due to the bias voltage change is 0.9 dB per millivolts if the mistuning effects can be negligible.

## REFERENCES

- [1] Aron, R. M., Bandwidth limitations and synthesis procedures for negative resistance and variable reactance amplifier, Tech. Rept. No 15, California Inst. of Tech., Pasadena, Calif., Aug 1960.
- [2] —, Gain bandwidth relations in negative resistance amplifiers, *Proc. IRE (Correspondence)*, vol 49, Jan 1961, pp 355-356.
- [3] Armstrong, L. D., Tunnel diodes for low noise microwave amplification, *Microwave J.*, vol 5, Aug 1962, 99-102.
- [4] Kurokawa, K., private communication.
- [5] Bode, H. W., *Network Analysis and Feedback Amplifier Design*, Princeton, N. J.: Van Nostrand, 1945, p 149.